Soil-Plant-Atmosphere Interactions

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Today's Lecture

- **Soil water balance**
- **Infiltration**
- **Evapotranspiration**
  - Surface energy balance
  - Penman-Monteith equation
  - Root water uptake

See Notes 4.pdf (self-check material on Notes 4 is also available in Moodle)

Soil water balance

EPFL

The hydrologic cycle

The total amount of water on Earth is invariable. At the same time water is continuously renewed while circulating between oceans, land and atmosphere.

All processes like *evaporation*, *precipitation*, *transpiration*, *infiltration*, *storage*, *runoff*, *groundwater flow*, which keep water in motion constitute the **hydrologic cycle**. Such processes are stimulated by solar energy, they take place simultaneously and, except for precipitation, continuously.

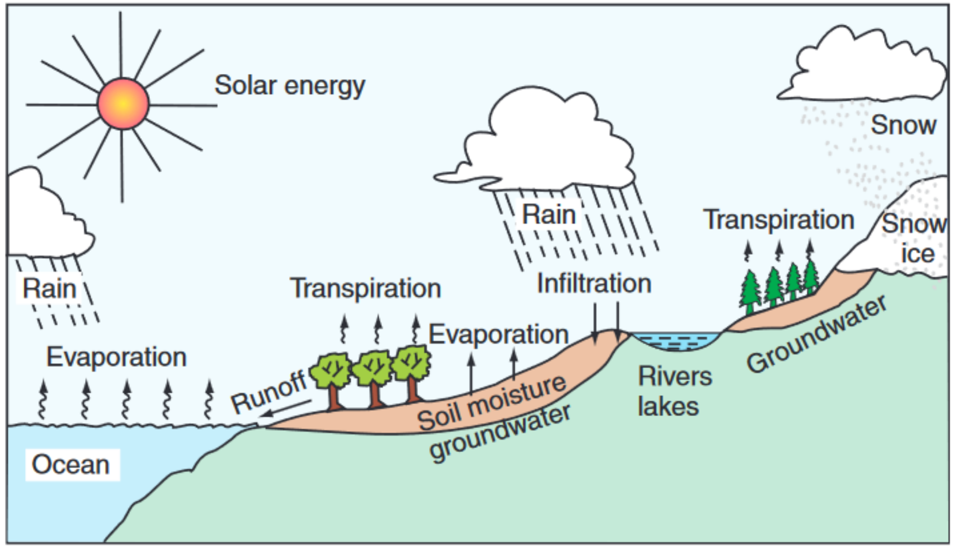


Fig. 20.2. The hydrologic cycle.

Hillel (2003)

Soil water balance

Mass balance (continuity equation):

$$\frac{\partial m}{\partial t} = IN - OUT \qquad [M/T]$$

If $\rho = const$ and $m = \rho V$:

$$\frac{\partial V}{\partial t} = in - out \qquad [L^3/T]$$

Water balance in the Root Zone:

$$\boxed{\frac{\partial S}{\partial t} = (P + I + U) - (R + D + E + Tr)}$$

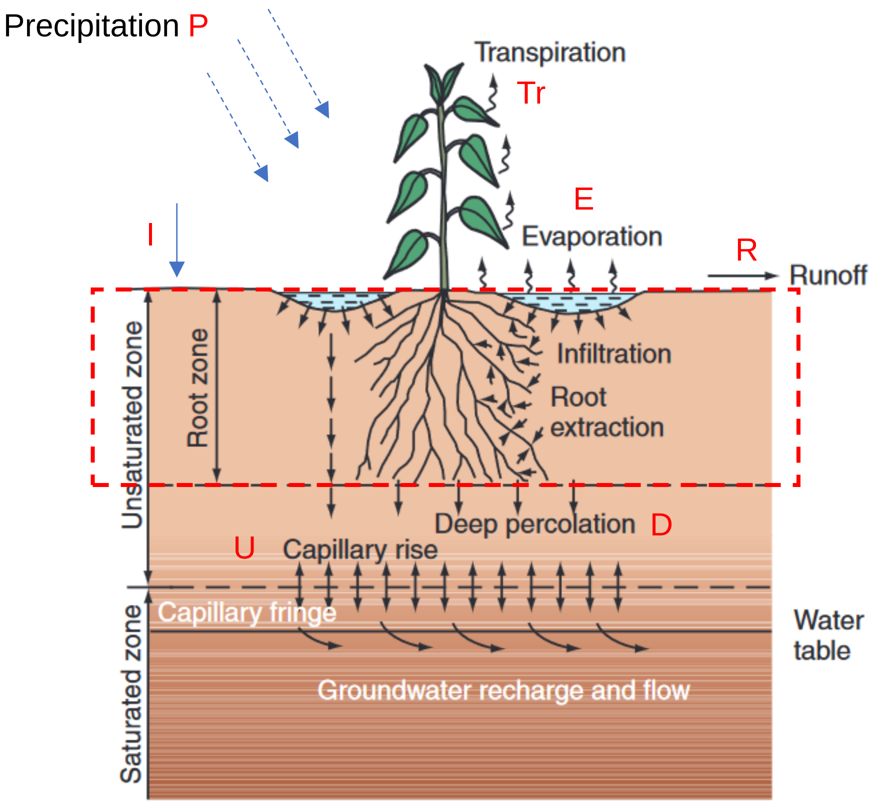


Fig. 20.1. The water balance of a root zone. Hillel (2003)

Soil water balance

$$\frac{\partial S}{\partial t} = \underbrace{(P + I + U)}_{\text{inputs}} - \underbrace{(R + D + E}_{\text{Outputs}} + \underbrace{Tr}_{\text{Internal sink}}) \quad [L/T]$$

P = precipitation (e.g., mm/d)

I = Irrigation

U = Capillary rise

R = Runoff

D = Drainage

E = Evaporation

$Tr = \int_0^z q_r \cdot dz$ = Transpiration (q_r= root water uptake)

$S = \int_0^z \theta dz$ = root-zone soil moisture storage

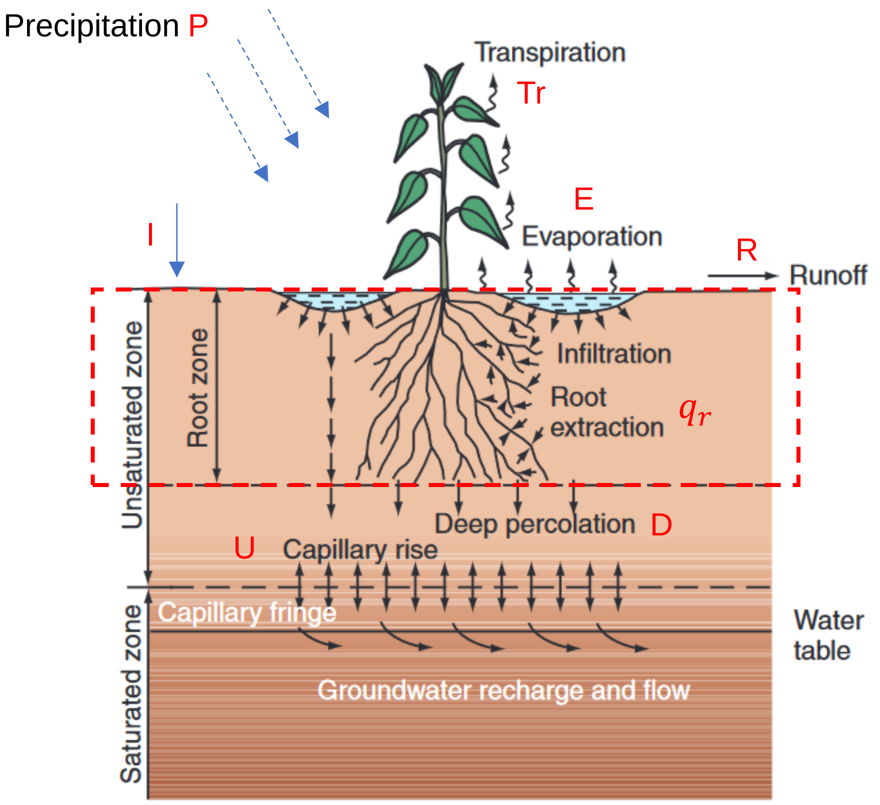


Fig. 20.1. The water balance of a root zone. Hillel (2003)

Note: units are volume over time per unit area $\rightarrow L^3\, T^{-1}\, L^{-2} \rightarrow L\, T^{-1}$

Infiltration

Infiltration is the term applied to the process of **water entry into the soil**, generally by downward flow through all or part of the soil surface.

The rate of this process, relative to the rate of *water supply*, determines how much water will enter the root zone and how much, if any, will **run off**

Kmec (2021)

Infiltration

If water is sprinkled over the soil surface at a steadily increasing rate, sooner or later the supply rate will exceed the soil's finite rate of absorption, and the excess will accrue over the soil surface or run off it

- **Water supply rate** (q_a)**:** e.g. rainfall rate or sprinkler irrigation
- **Infiltration rate** (q_i)**:** the flux of water flowing into the profile per unit of soil surface area
- **infiltration capacity** (or **infiltrability,** i)**:** the maximum rate at which water can be absorbed by a given soil per unit area under given conditions

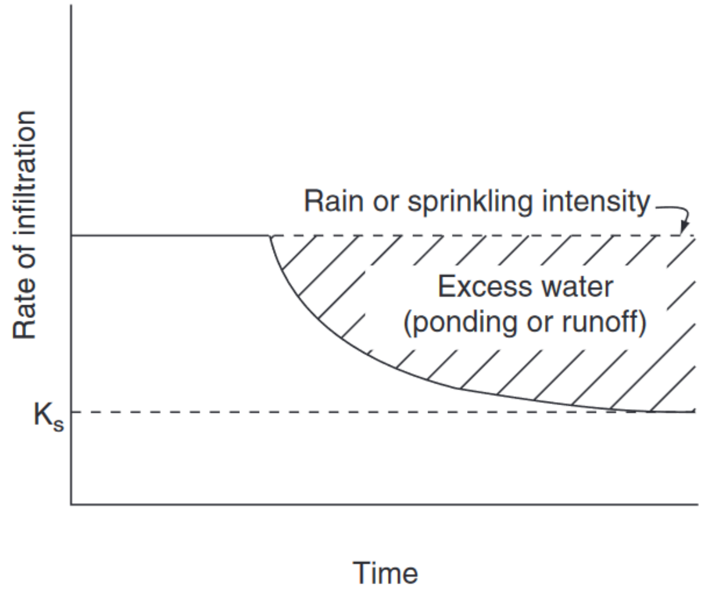


Fig. 14.1. Time dependence of infiltration rate under rainfall of constant intensity that is lower than the initial value but higher than the final value of soil infiltrability.

Hillel (2003)

Infiltration

- As long as the rate of water delivery to the surface is smaller than the soil's infiltrability, water infiltrates as fast as it arrives and the process is **supply controlled** (or flux controlled): $q_a < i \rightarrow q_i = q_a$

- If the delivery rate exceeds the soil's infiltrability, the latter determines the actual infiltration rate, and thus the process becomes **soil controlled**: $q_a > i \rightarrow q_i = i$
  - when $q_a = i$, the surface becomes ponded
  - the time at which this occur is called **ponding time**

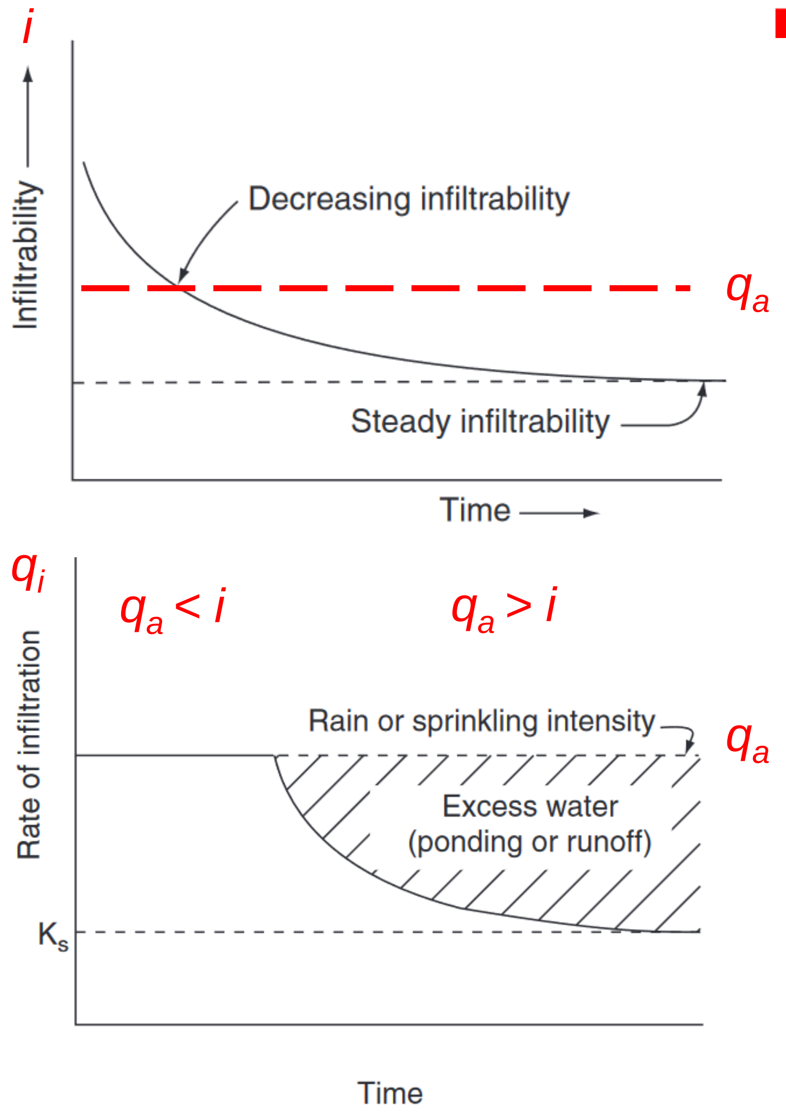


Fig. 14.1. Time dependence of infiltration rate under rainfall of constant intensity that is lower than the initial value but higher than the final value of soil infiltrability.

Hillel (2003)

Infiltration

Soil infiltrability is relatively high in the early stages of infiltration, particularly where the soil is initially quite dry, but it tends to decrease and eventually to approach asymptotically a constant rate called the **steady-state infiltrability** (→ corresponding to the Saturated Hydraulic Conductivity!)

This **decline** of infiltrability primarily results from the **decrease in the matric suction gradient**, which occurs inevitably as infiltration proceeds

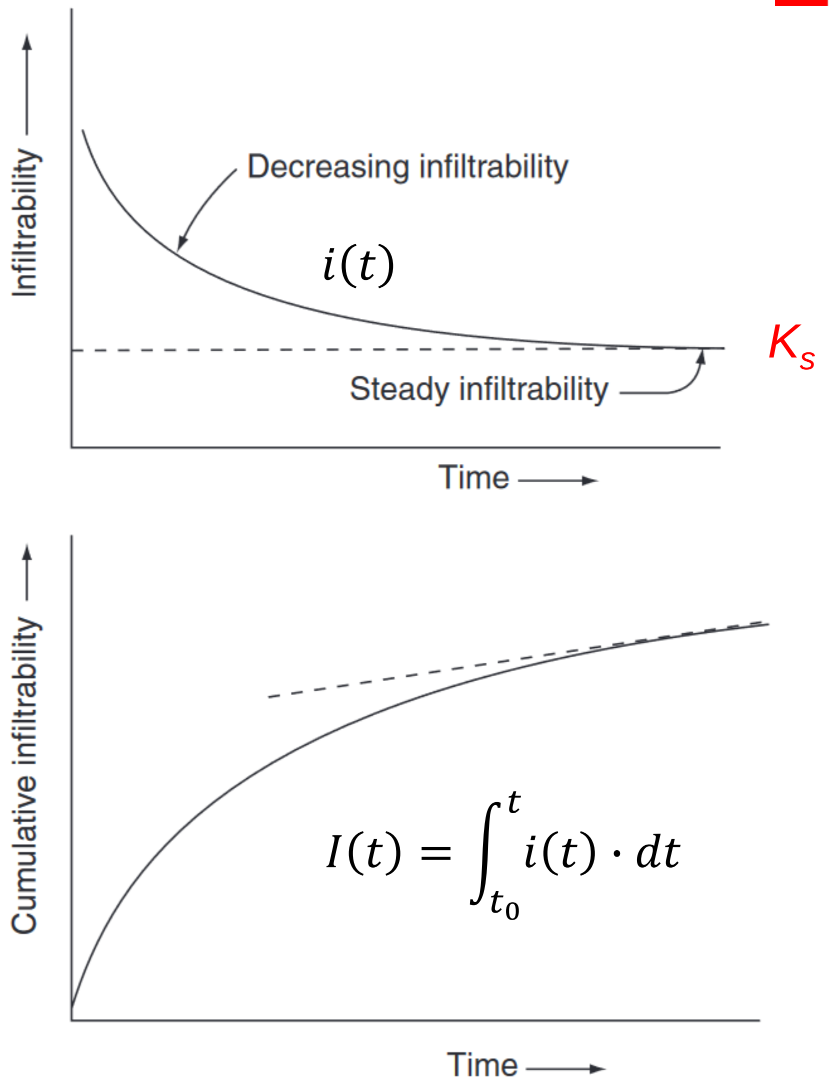
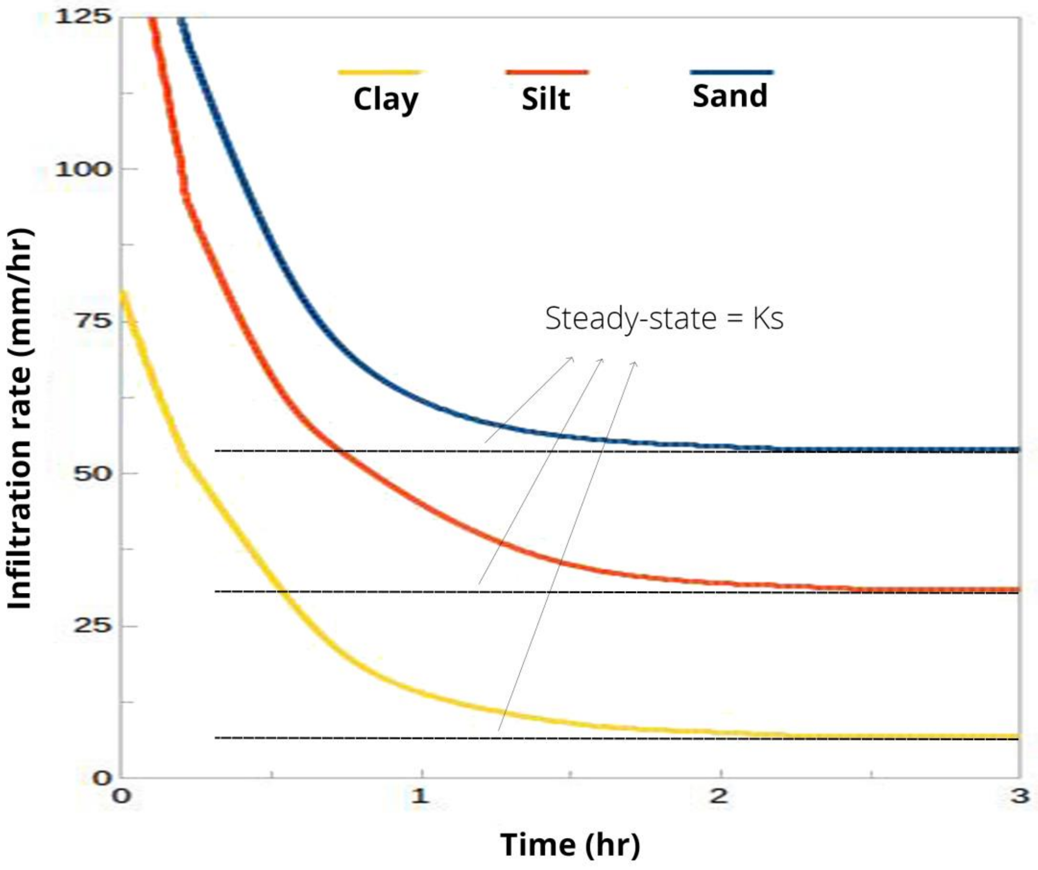


Fig. 14.2. Time dependence of infiltrability and of cumulative infiltration under shallow ponding.

Hillel (2003)

Infiltration

Soil infiltrability and its variation with time are known to depend on the *initial wetness* and suction as well as on the *texture*, *structure*, and *uniformity* (or layering sequence) of the profile.



Source: Google Images

Infiltration

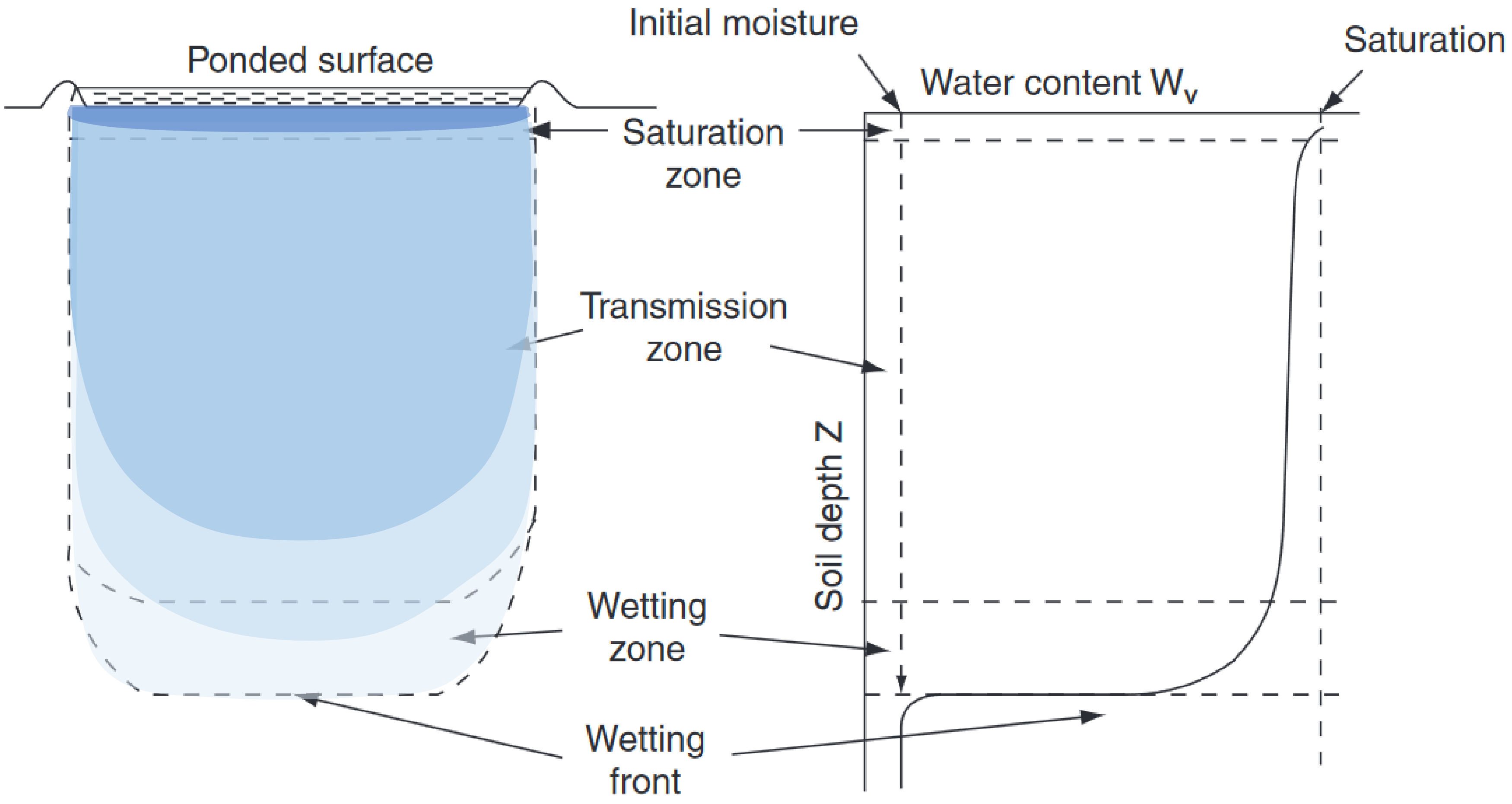


Fig. 14.3. The infiltration moisture profile. At left, a schematic section of the profile; at right, the curve of water content versus depth. The common occurrence of a saturation zone as distinct from the transmission zone may result from the structural instability of the surface zone.

Hillel (2003)

Infiltration

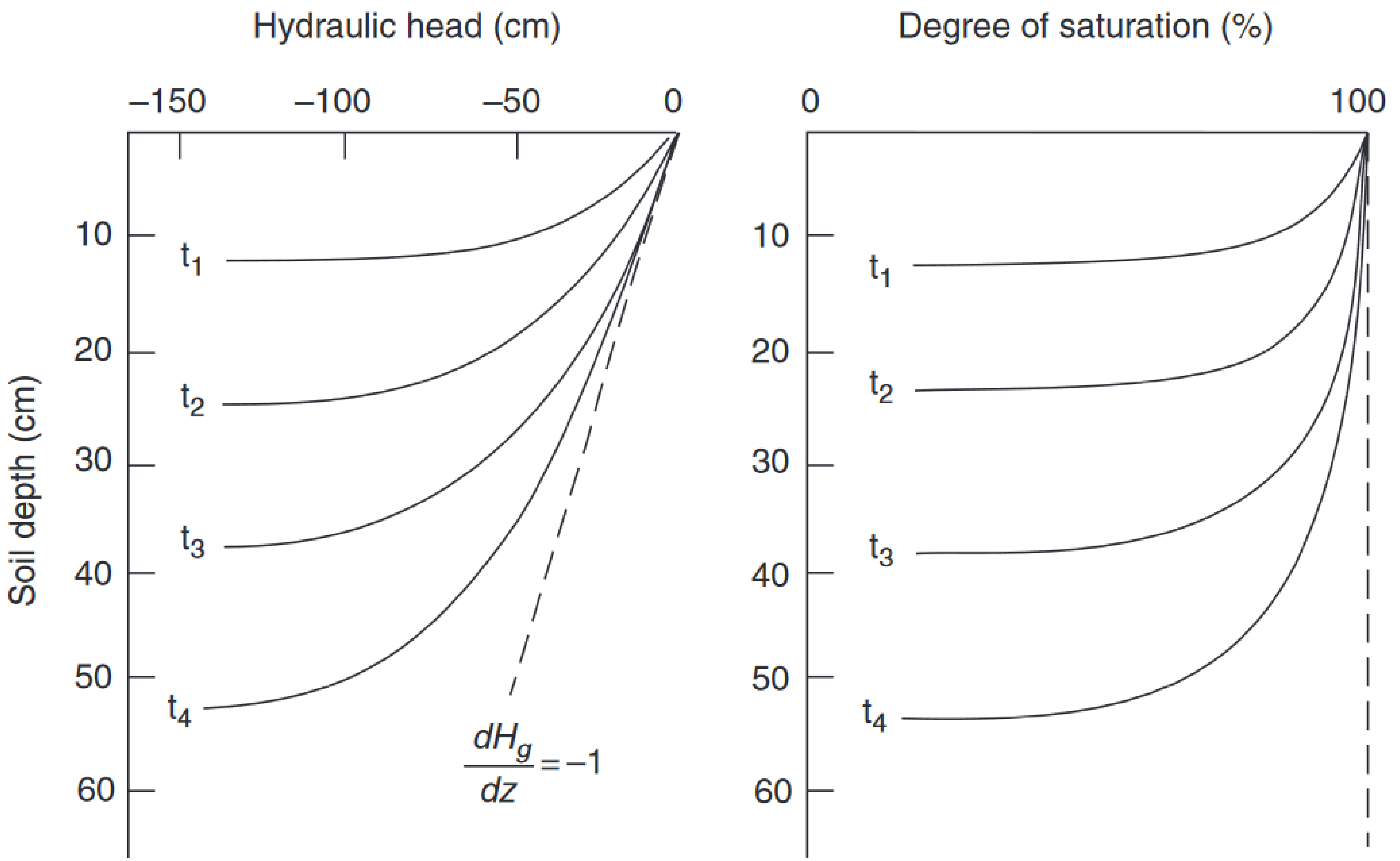


Fig. 14.4. Water-content profiles (at right) and hydraulic-head profiles (at left) at successive times (t_1,t_2,t_3,t_4) during infiltration into a uniform soil ponded at the surface. The dH_g/dz value is the gravitational head gradient. In this figure the possible existence of a saturation zone distinct from the transmission zone is disregarded.

Hillel (2003)

Empirical infiltration equations

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The Lewis (Kostiakov) Equation

One of the most widely used empirical expressions was originally proposed by Lewis (1937) but was erroneously attributed to Kostiakov (1932; see discussion by Swartzendruber, 1993):

$$I(t) = kt^a \quad \Rightarrow \quad i(t) = \frac{dI}{dt} = akt^{a-1}$$

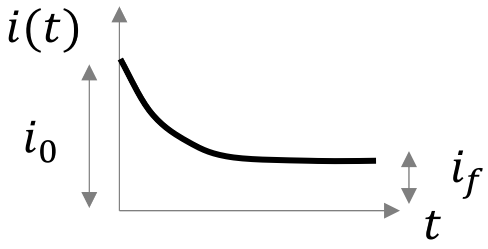
cumulative infiltration — Infiltration rate

The Horton Equation

Horton (1940) proposed another empirical equation based on an exponential form:

$$I(t) = i_f t + \frac{i_0 - i_f}{\delta}\left[1 - e^{-\delta t}\right] \quad \Rightarrow \quad i(t) = i_f + \left(i_0 - i_f\right)e^{-\delta t}$$

Where i_0 = initial infiltration capacity
i_f = final infiltration capacity



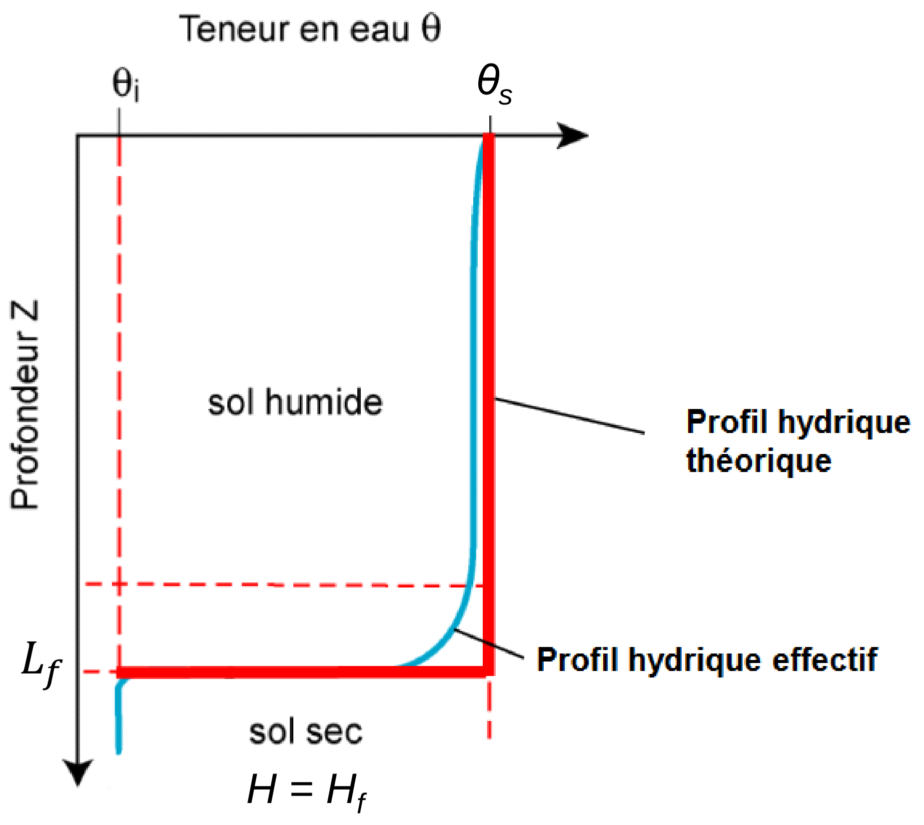
Physically-based infiltration equations

The Green-Ampt Approximation

The basic assumptions of Green and Ampt (1911) are:

- a distinct wetting front exists such that the water content behind it (θ_0) remains constant and abruptly changes to initial water content (θ_i) ahead of the wetting front;
- the soil in the wetted region has constant properties (θ_0, K, and h_0);
- the matric potential at the wetting front is constant and equal to H_f

These assumptions simplify the flow equation, making it amenable to analytical solution.



Physically-based infiltration equations

The Green-Ampt Approximation (derivation)

For **horizontal** infiltration, a Darcy-type equation can be applied as:

$$i = \frac{dI}{dt} = K\frac{H_0 - H_f}{L_f}$$

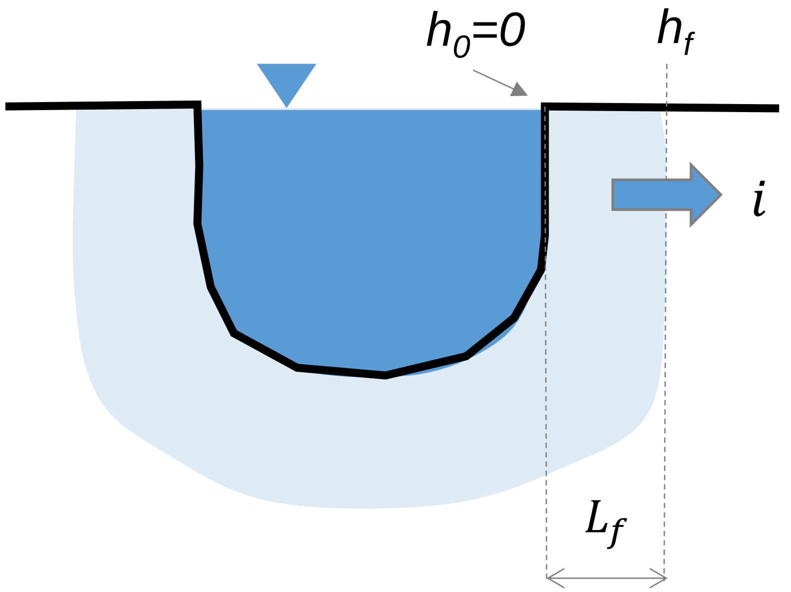
$$= K\frac{h_0 - h_f}{L_f}$$

$$= K\frac{\Delta h}{L_f} = -K\frac{h_f}{L_f}$$

$h_0 = 0$

i.e., p_{atm} assuming that the ponding depth is negligible

Horizontal infiltration



Physically-based infiltration equations

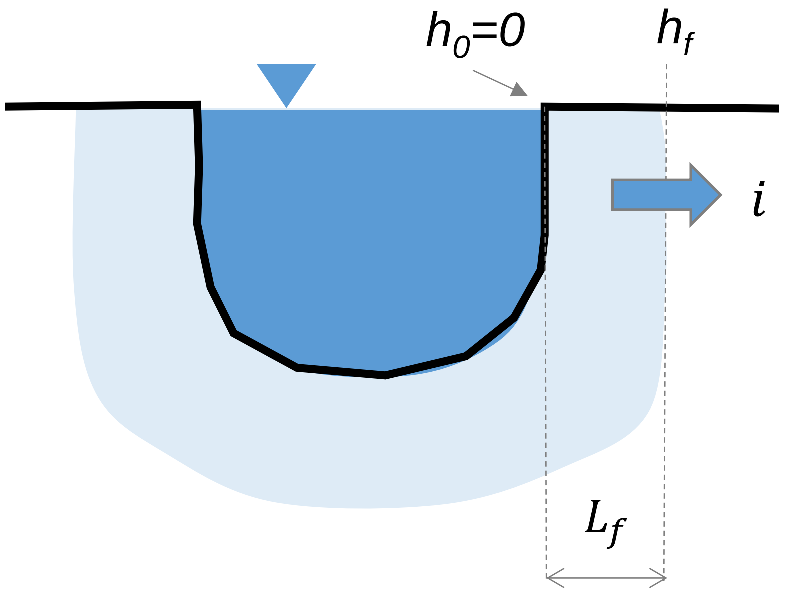
The Green-Ampt Approximation (derivation)

Since a uniformly wetted zone is assumed to extend all the way to the wetting front, it follows that the cumulative infiltration *I* should be equal to:

$$I = \Delta\theta \cdot L_f = (\theta_0 - \theta_f)L_f$$

Note: a special case is when $\theta_f = 0$ and $\theta_0 = \theta_s$ then $\Delta\theta = n$ (porosity)

Horizontal infiltration



$$\Rightarrow \quad i = \frac{dI}{dt} = \Delta\theta \frac{dL_f}{dt}$$

The infiltration rate is equal to the rate of advance of the **wetting front** times the change in soil water storage

Physically-based infiltration equations

EPFL

The Green-Ampt Approximation (derivation)

Darcy

$$i = \Delta\theta \frac{dL_f}{dt} = K \frac{\Delta h}{L_f}$$

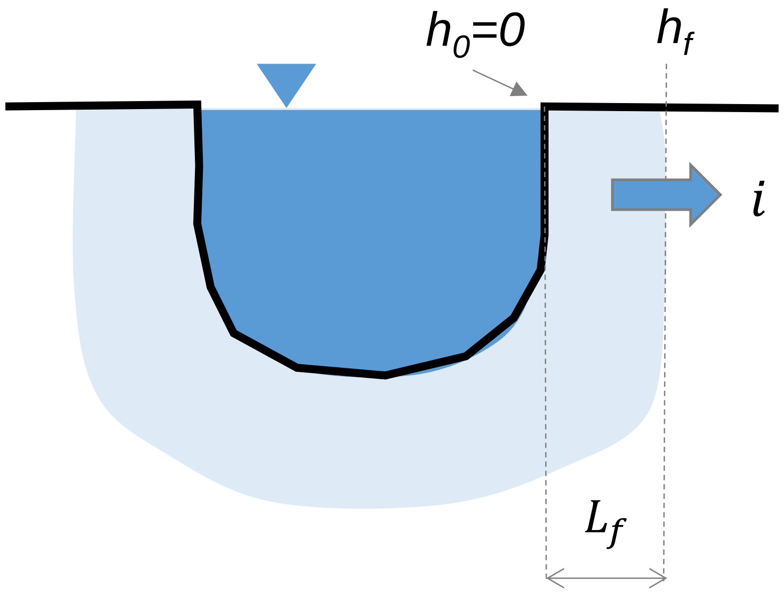
Upon integration:

$$\int_0^{L_f} L_f \cdot dL_f = K \frac{\Delta h}{\Delta\theta} \int_0^{t} dt$$

$$\Rightarrow \quad \frac{L_f^2}{2} = K \frac{\Delta h}{\Delta\theta} t = Dt \quad \Rightarrow \quad L_f = \sqrt{2Dt}$$

Effective Diffusivity

Horizontal infiltration



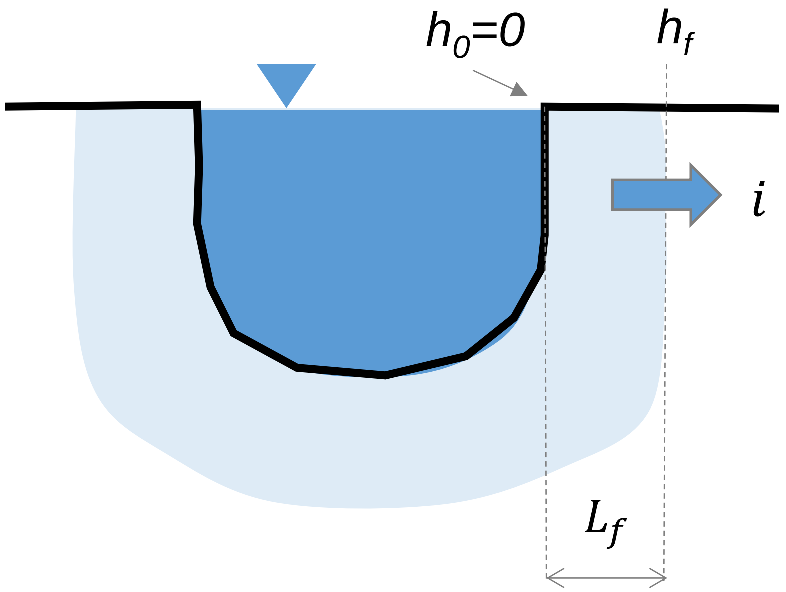
Physically-based infiltration equations

The Green-Ampt Approximation (derivation)

$$\Rightarrow \quad I = \Delta\theta \cdot L_f = \Delta\theta\sqrt{2Dt} \sim t^{1/2}$$

$$i = \Delta\theta\sqrt{\frac{D}{2t}} \sim t^{-1/2}$$

Horizontal infiltration



Hence, the lenght of the wetting front is proportional to $t^{1/2}$, and the infiltration rate to $t^{-1/2}$

Physically-based infiltration equations

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The Green-Ampt Approximation (derivation)

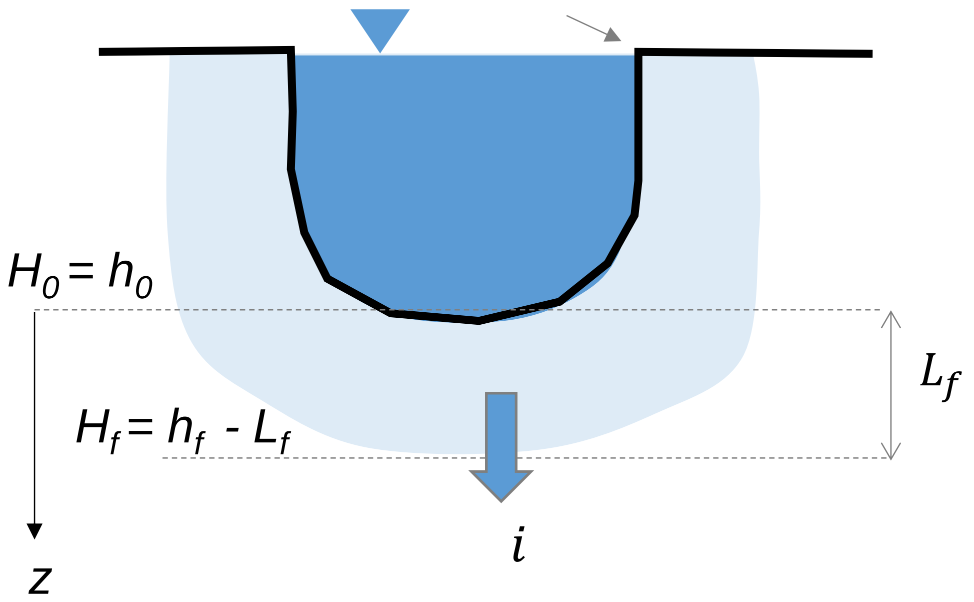
For **vertical** infiltration, the Green-Ampt approach gives:

$$i = \frac{dI}{dt} = \Delta\theta \frac{dL_f}{dt}$$

$$= K \frac{H_0 - H_f}{L_f}$$

$$= K \frac{h_0 - h_f + L_f}{L_f}$$

Vertical infiltration



Upon integration: $$\int_0^{L_f} \frac{L_f}{\Delta h + L_f} \cdot dL_f = \frac{K}{\Delta\theta} \int_0^t dt$$

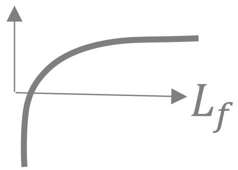
$$\Rightarrow \boxed{L_f - \Delta h \cdot \ln\left[1 + \frac{L_f}{\Delta h}\right] = \frac{Kt}{\Delta\theta}}$$

Physically-based infiltration equations

The Green-Ampt Approximation (derivation)

It follows that:

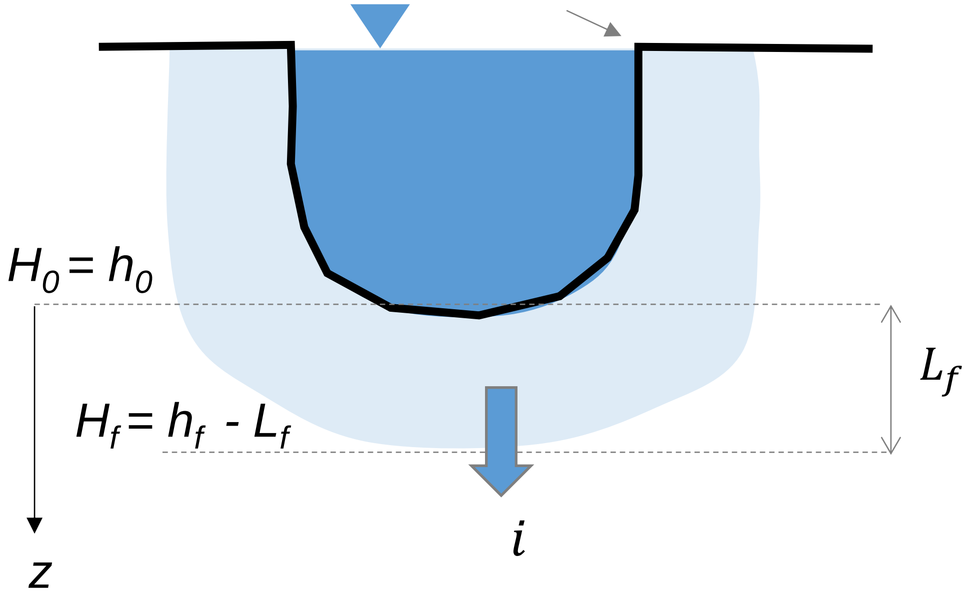
$$L_f = \frac{Kt}{\Delta\theta} + \Delta h \cdot \ln\left[1 + \frac{L_f}{\Delta h}\right]$$



For short times this solution converges to the solution of the horizontal case, while for long infiltration periods:

$$\Rightarrow \quad L_f \sim \frac{Kt}{\Delta\theta} + \delta \quad \Rightarrow \quad I \sim Kt + \delta \quad \Rightarrow \quad \boxed{i \sim K}$$

Vertical infiltration



Physically-based infiltration equations

Philip's solution

Philip (1957, 1969) presented the first analytical solution to the Richards equation for vertical and horizontal infiltration.

Horizontal case

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x}\left(K\frac{\partial H}{\partial x}\right) \xrightarrow{\text{Using the Diffusivity eq.}} \frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x}\left(K\frac{\partial\theta}{\partial x}\right)$$

IC & BC:

$$\left.\begin{aligned}\theta(t=0, x\geq 0) &= \theta_i\\ \theta(t>0, x=0) &= \theta_0\end{aligned}\right\}$$

Using the Boltzmann Transformation (see Hillel 2003)

$$\boxed{I = S\cdot t^{\frac{1}{2}} \quad \text{and} \quad i = \frac{1}{2}S\cdot t^{-\frac{1}{2}}}$$

$S = S(\theta_i, \theta_0)$ is the **sorptivity** [L T$^{-1/2}$]

Physically-based infiltration equations

Philip's solution

Philip (1957, 1969) presented the first analytical solution to the Richards equation for vertical and horizontal infiltration.

Vertical case

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(K \frac{\partial h}{\partial z}\right) + \frac{\partial K}{\partial z} \xrightarrow{\text{Using the Diffusivity eq.}} \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(D \frac{\partial \theta}{\partial z}\right) + \frac{\partial K}{\partial z}$$

IC & BC:

$$\left.\begin{aligned} \theta(t = 0, z \geq 0) &= \theta_i \\ \theta(t > 0, z = 0) &= \theta_0 \end{aligned}\right\}$$

Using a power series (see Hillel 2003)

$$\boxed{\begin{aligned} I(t) &= S \cdot t^{\frac{1}{2}} + A_1 \cdot t^{\frac{2}{2}} + A_2 \cdot t^{\frac{3}{2}} + \cdots \\ i(t) &= \frac{1}{2} S \cdot t^{-\frac{1}{2}} + A_1 \end{aligned}}$$

Physically-based infiltration equations

Philip's solution

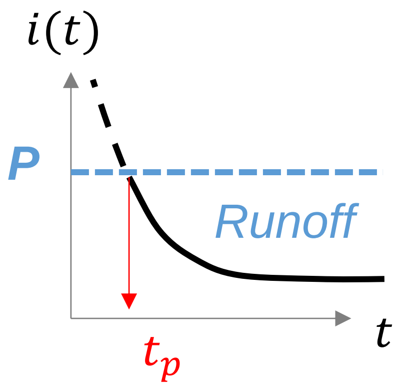
$$i(t) = \frac{1}{2} S \cdot t^{-\frac{1}{2}} + A_1$$

Contribution of sorptive forces or (relatively) dry soil

Contribution of gravity

For flux-limited infiltration rate such as low intensity rainfall, P, we may estimate whether and when surface **runoff** will occur by calculating the **time to ponding** t_p, i.e. the time at which $i = P$:

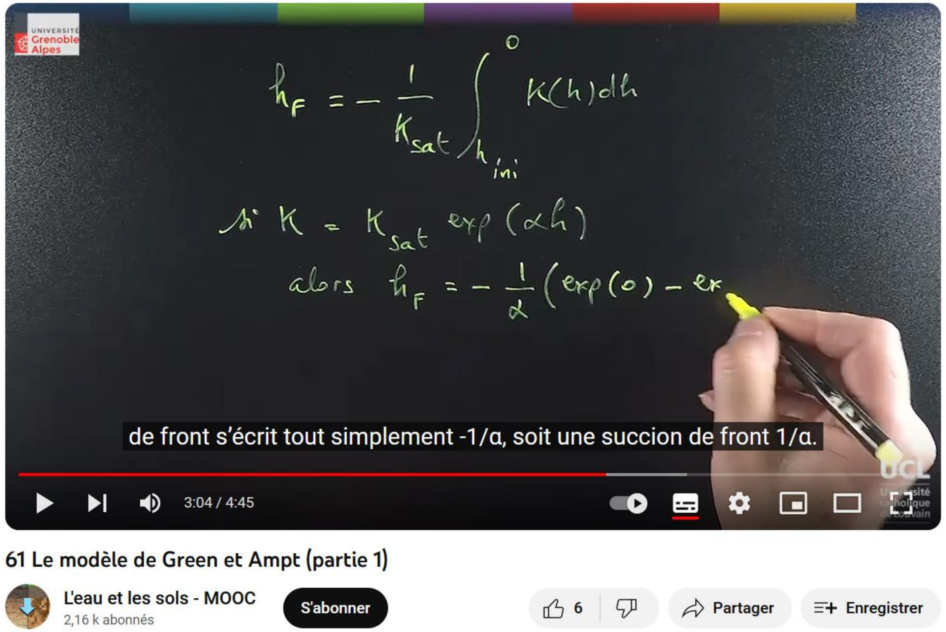
$$t_p = \frac{S^2}{4(P - A_1)^2}$$



Useful links

Derivation of the Green and Ampt infiltration formula:

- [PART 1](Youtube)(Youtube)
- PART 2 (Youtube)



Surface energy balance & Evapotraspiration

Soil water balance: $$\frac{\partial S}{\partial t} = P - R - D - ET$$ $$ET = E + Tr =$$ **Evapotranspiration**

ET is the combined processes which move water from the Earth's surface into the atmosphere. It covers both **water evaporation *E*** (movement of water to the air directly from soil, canopies, and water bodies) and **transpiration *Tr*** (evaporation that occurs through the stomata, or openings, in plant leaves).

ET is primarily controlled by:

- the amount of water present in the soil and atmosphere (→ **soil water balance**)
- the amount of energy available (→ **surface energy balance**)

Source: Google Images

Surface energy balance & Evapotraspiration

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Surface Energy balance

Solar radiation, energy in the form of electromagnetic waves emitted by the sun, is the main source of energy for processes near the earth's surface. Solar radiation is the driving force behind the hydrologic cycle, including transpiration by plants and evaporation of water from the soil and from bodies of water.

The **net radiation** (R_n) absorbed by a surface is:

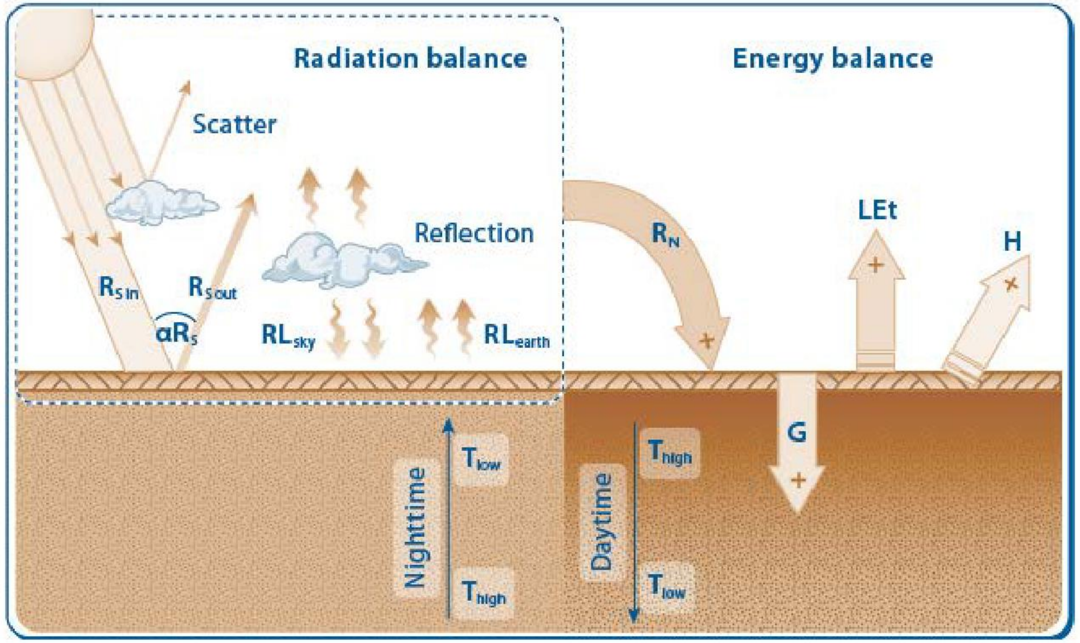

Fig.3-3: Schematic representation of radiation and energy balance components.

Source: Or, Tuller, & Wraith, 1994-2018

Short-wave **Long-wave**

$$\boxed{R_n = (R_s^{\downarrow} - R_s^{\uparrow}) + (R_L^{\downarrow} - R_L^{\uparrow})}$$

Incoming (Emitted from the sun)

Reflected from surface

Incoming sky radiation

Outgoing (Emitted from the Earth)

Surface energy balance & Evapotraspiration

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Surface Energy balance

The net radiation impinging on the earth's surface is then partitioned into several components. Part of the energy may be transformed into heat which warms the soil, plants, and the atmosphere; another part may be used by plants for photosynthesis. A major part of R_n is used for evaporating water in the combined process of evapotranspiration.
The energy balance on a field-scale surface is given by:

$$\boxed{R_n^{\downarrow} = H^{\uparrow} - LE^{\uparrow} - G^{\downarrow}}$$

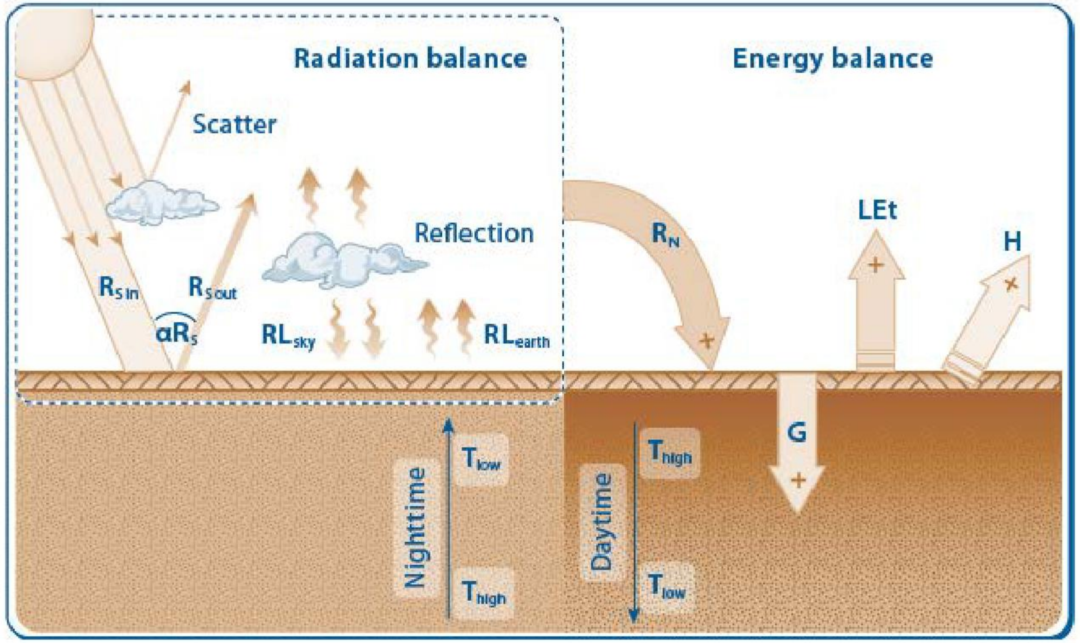


Fig.3-3: Schematic representation of radiation and energy balance components.

Source: Or, Tuller, & Wraith, 1994-2018

Where:

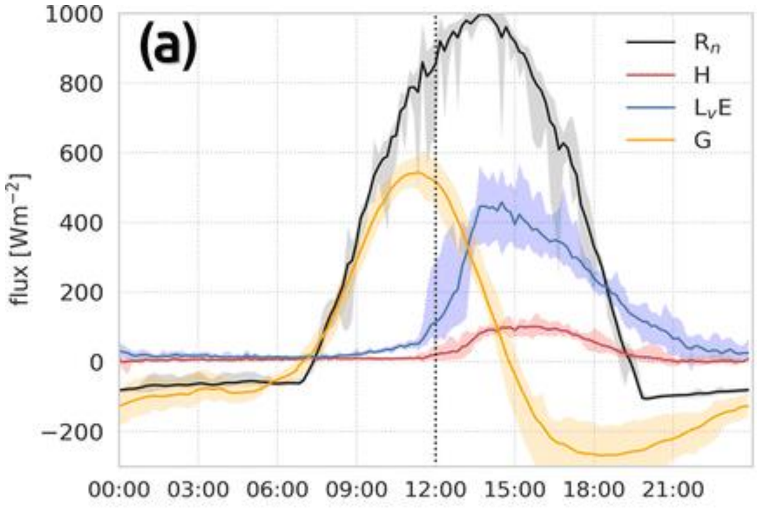
- H = energy utilized in heating the air (known as the **sensible heat flux, W m^{-2}**),
- G = is the energy utilized in heating the soil (**soil heat flux, W m^{-2}**)
- $LE = \lambda \cdot ET$ = Energy for evaporating water (**latent heat flux, W m^{-2}**)

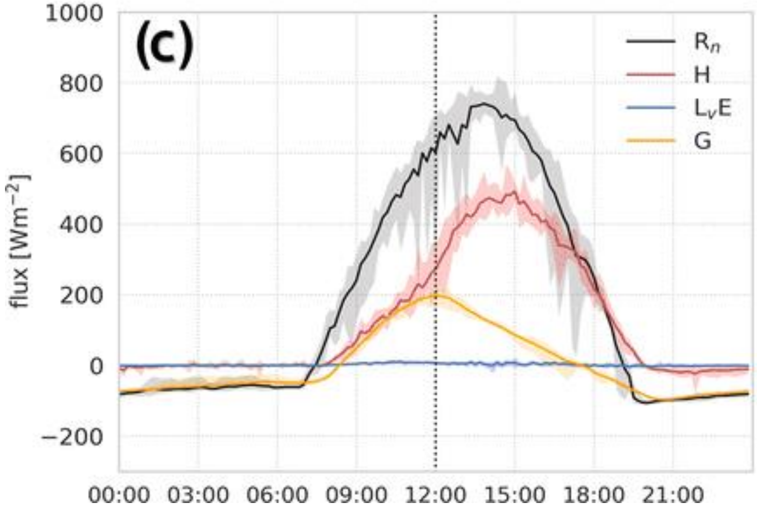
Latent heat of vaporization = 2.449 MJ/kg

Surface energy balance & Evapotraspiration

When the soil surface is **wet** or covered with well-watered plants, most of the energy from R_N goes to LE.

As the soil becomes **drier**, a larger part of the incoming energy goes to heating the air (H) and the soil (G).





Lobos et al. (2021)

Surface energy balance & Evapotraspiration

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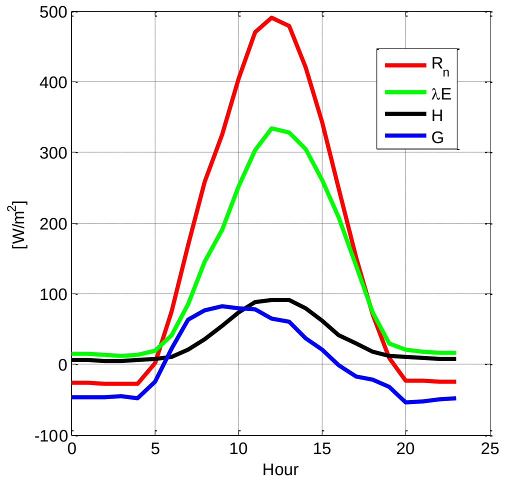

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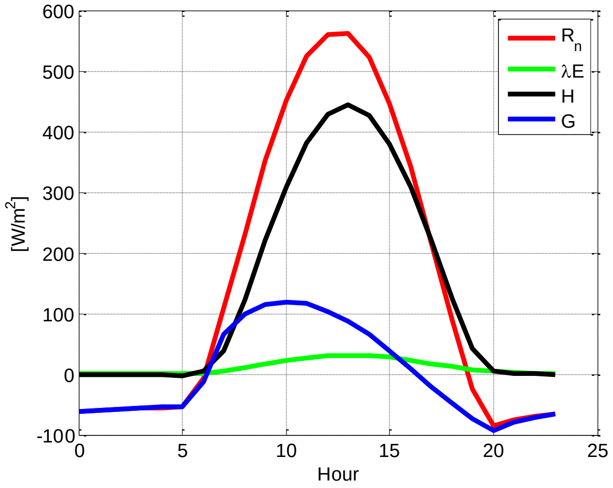

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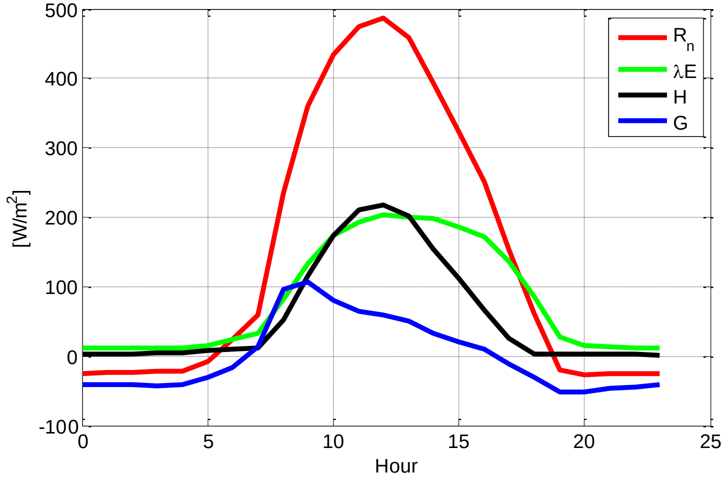
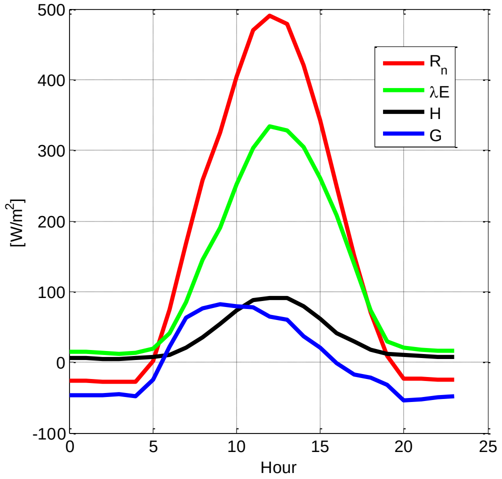


Can you guess where these measurements were taken?

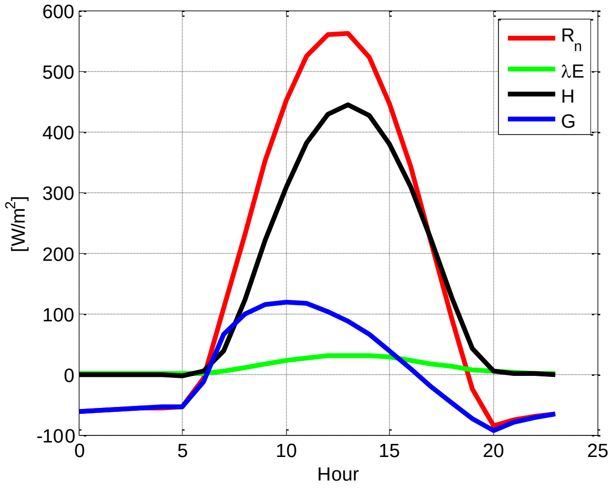
(**Hint**: natural ecosystems, no water bodies)

Surface energy balance & Evapotraspiration

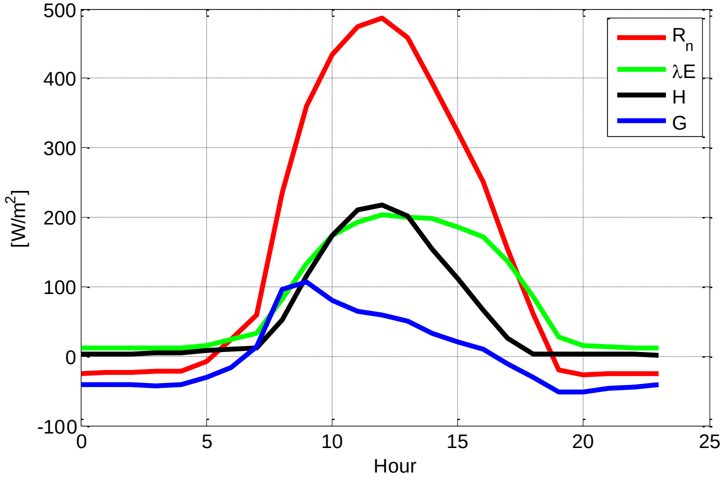
Chamau (CH) Grassland – June



Sacramento (CA) Grassland – June



Davos (CH) – Evergreen Forest - July



Surface energy balance & Evapotraspiration

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Estimation of ET

- One of the methods for estimating ET is the **water balance method**: it is based on measurements of rainfall, irrigation, runoff, changes in soil water content, and deep percolation, to estimate the unknown value of ET
- Climatologically-based methods include:
  - Aerodynamic profile methods - based on mass transfer of vapor and heat
  - Energy balance methods – directly measuring or estimating all attributes other than ET, similar to the water balance
  - **Combination methods** (aerodynamic + energy) → e.g., **Penman-Monteith equation**
  - **Empirical methods** - based on empirical relationships, usually applicable only to monthly or seasonal ET estimates → e.g., **Thornthwaite Method, Turc's formula**

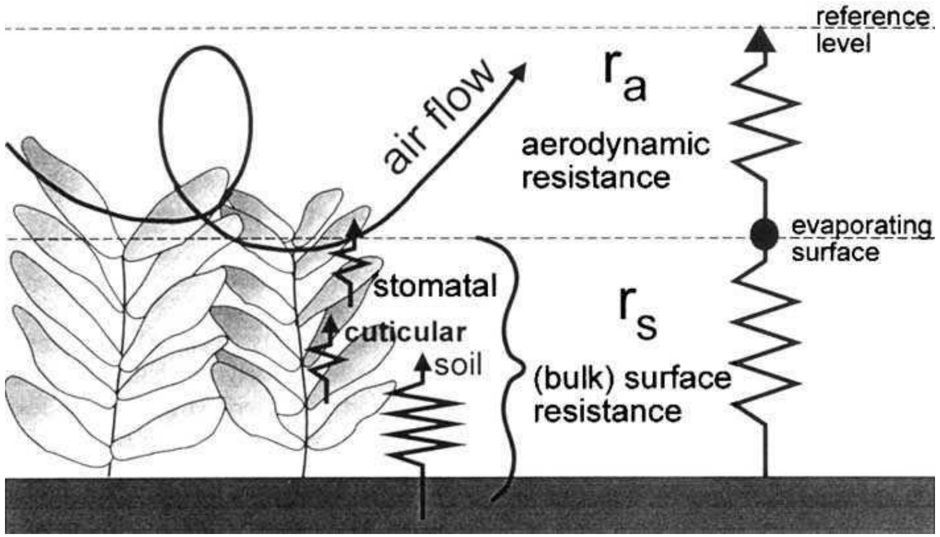
$$\frac{\partial S}{\partial t} = P - R - D - ET$$

$$R_n = H + \lambda ET + G$$

Surface energy balance & Evapotraspiration

Penman-Monteith equation

$$LE_p = \frac{\Delta(R_n - G) + \rho_a c_p \dfrac{(e_a^* - e_a)}{r_a}}{\Delta + \gamma\left(1 + \dfrac{r_s}{r_a}\right)}$$

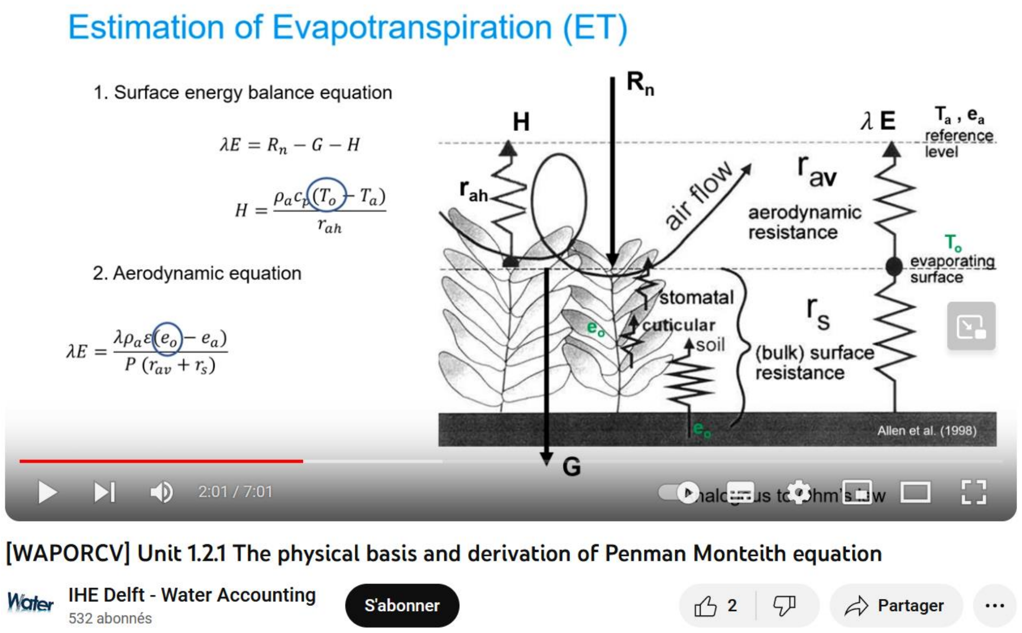


where R_n is the net radiation, G is the soil heat flux, $(e_s - e_a)$ represents the vapour pressure deficit of the air, r_a is the mean air density at constant pressure, c_p is the specific heat of the air, Δ represents the slope of the saturation vapour pressure temperature relationship, γ is the psychrometric constant, and r_s and r_a are the (bulk) surface and aerodynamic resistances

Note: the main use of Penman's equation is for estimation of **potential evapotranspiration** ($ET_p = LE_0/\lambda$) where water is not limiting and the resistance to flow of water vapor is negligible.

Surface energy balance & Evapotraspiration

Useful Link: [Derivation of Penman Monteith equation](#) (Youtube)



Surface energy balance & Evapotraspiration

Empirical methods (examples)

The **Thornthwaite Method (1948)** estimates ET_p [cm/month] from monthly mean temperature $\bar{T}$ [°C] and a monthy heat index I:

$$ET_p = 1.6\left(\frac{10\bar{T}}{I}\right)$$

The **Turc's formula (1961)** estimates ET_p from monthly mean temperature $\bar{T}$ [°C] and global solar radiation R_s [cal cm^{-2} J^{-1}]

$$ET_p = 0.4(R_s + 50)\frac{\bar{T}}{\bar{T} + 15}$$

Surface energy balance & Evapotraspiration

Given the multiplicity of factors influencing evapotranspiration (e.g., water availability, vegetation cover, wind speed), the following concepts have been introduced to estimate crop water requirements (see FAO Guidelines):

- **Potential Evapotranspiration (PET or ET_p)** is defined as the amount of evaporation that would occur if a sufficient water source were available. If the actual evapotranspiration is considered the net result of atmospheric demand for moisture from a surface and the ability of the surface to supply moisture, then PET is a measure of the demand side.

- **Reference evapotranspiration (ET_0)** is a representation of the environmental demand for evapotranspiration and represents the evapotranspiration rate of a short green crop (grass), completely shading the ground, of uniform height and with adequate water status in the soil profile. It is a reflection of the energy available to evaporate water, and of the wind available to transport the water vapour from the ground up into the lower atmosphere.

- **Actual evapotranspiration (ET)** is equal to potential evapotranspiration when there is ample water but, in general, **$ET < ET_p$**

Surface energy balance & Evapotraspiration

Reference evapotranspiration (short grass):

$$ET_0 = ET_p(reference\ grass)$$

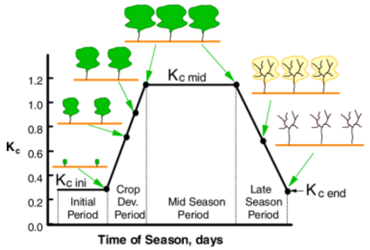
Penman-Monteith equation using:

- r_a = 208/u(z=2m) [s/m]
- r_s = 70 [s/m]

Crop evapotranspiration under standard conditions:

$$ET_c = K_C \cdot ET_0$$

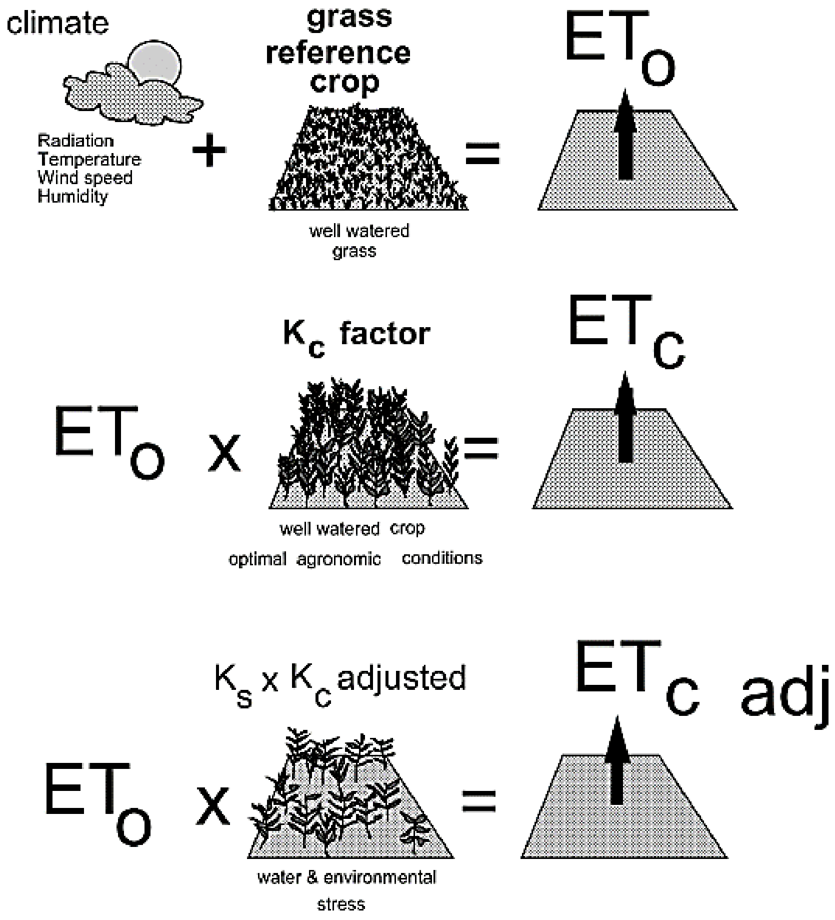
Crop coefficient



Crop evapotranspiration under non-standard condition:

$$ET_{c,adj} = K_S \cdot K_C \cdot ET_0$$

Stress coefficient (e.g., water stress, pests, soil salinity, low fertility)



https://www.fao.org/3/x0490e/x0490e04.htm

Surface energy balance & Evapotraspiration

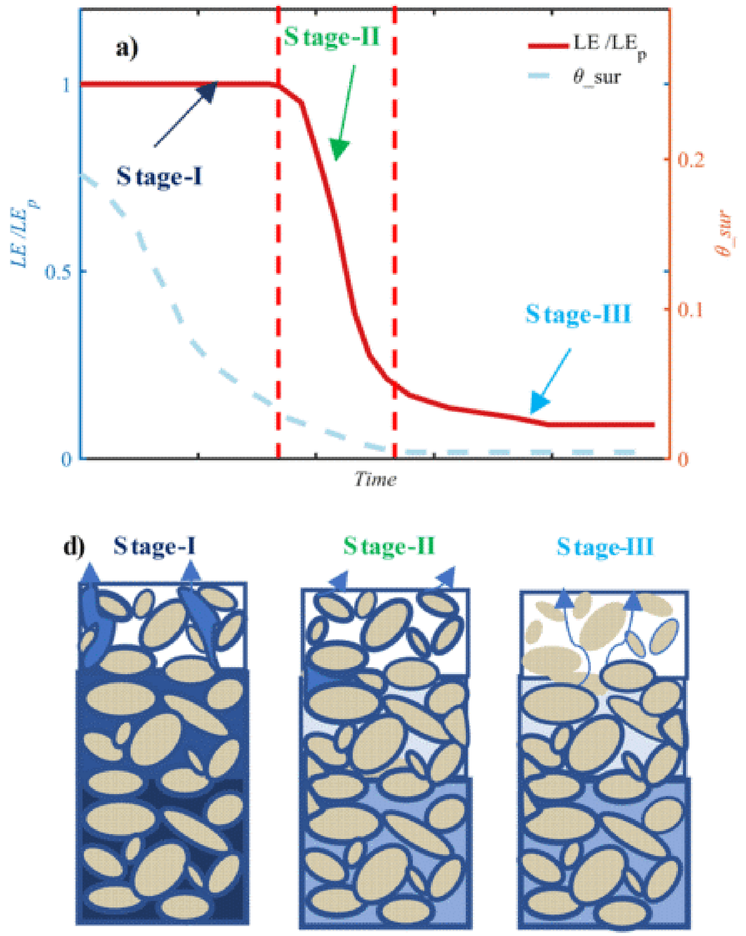
Soil Evaporation & Transpiration

Evaporation from initially moist bare soil takes place in 3 stages:

I. steady-state phase: evaporation controlled by evaporative demand (available energy, relative humidity, wind) $E = E_p$ Potential evaporation

II. decreasing rate phase: evaporation gradually decreases, as the soil's capacity to supplyto supply water to the surface $E = \frac{\alpha}{2} t^{-1/2}$ α=constant t=time from start of phase II

III. slow phase: low evaporation, mainly due to diffusion of water vapour from the soil



Wang et al. (2019), WRR

Surface energy balance & Evapotraspiration

Soil Evaporation & Transpiration

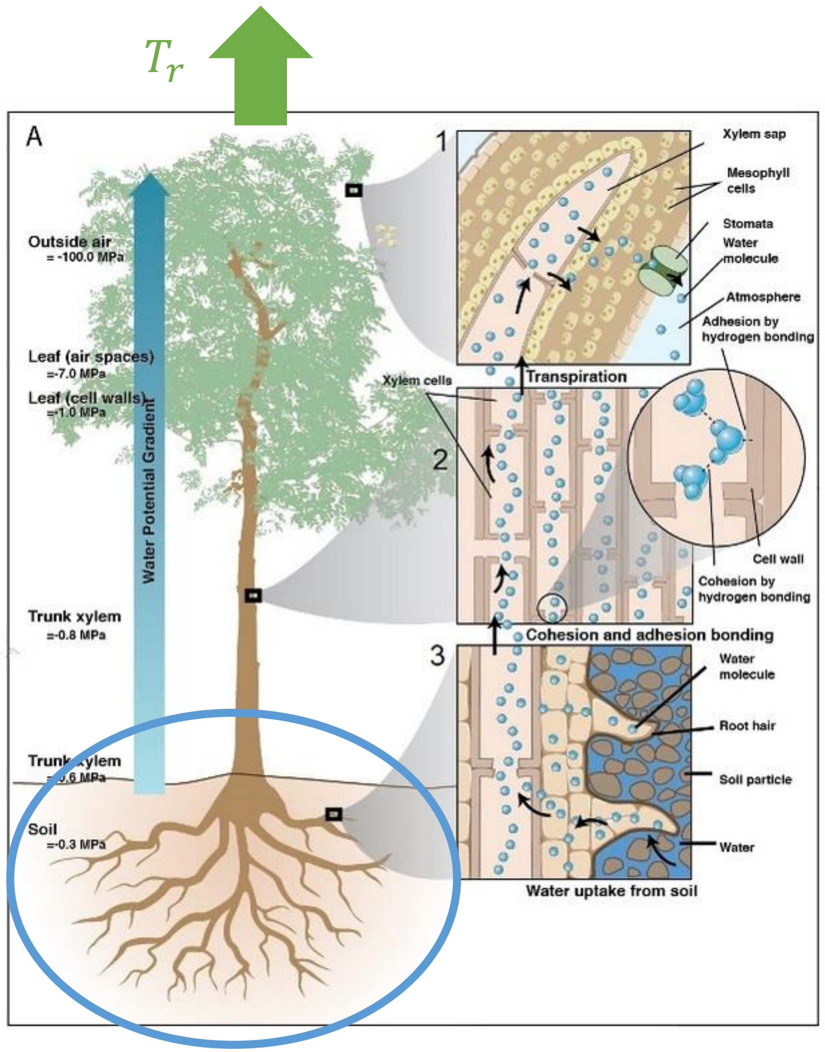
Transpiration is the process by which plants lose water in the form of vapour through the stomata.

Plants control the flow of water exchanged with the air by opening or closingc their stomata; if soil water availability is reduced or in the presence of other stress factors (high VPD, poor health, nutrient deficit, etc.), plants reduce their water consumption (hence, *actual transpiration* < *potential transpiration*).

Note: water transpired by leaves is up-taken from the soil by the rooting system, hence:

Transpiration: $$T_r = \int_{-z_r}^{0} Q(z) \cdot dz$$

Root water uptake (at depth z)



See: [Water Uptake and Transport in Vascular Plants]

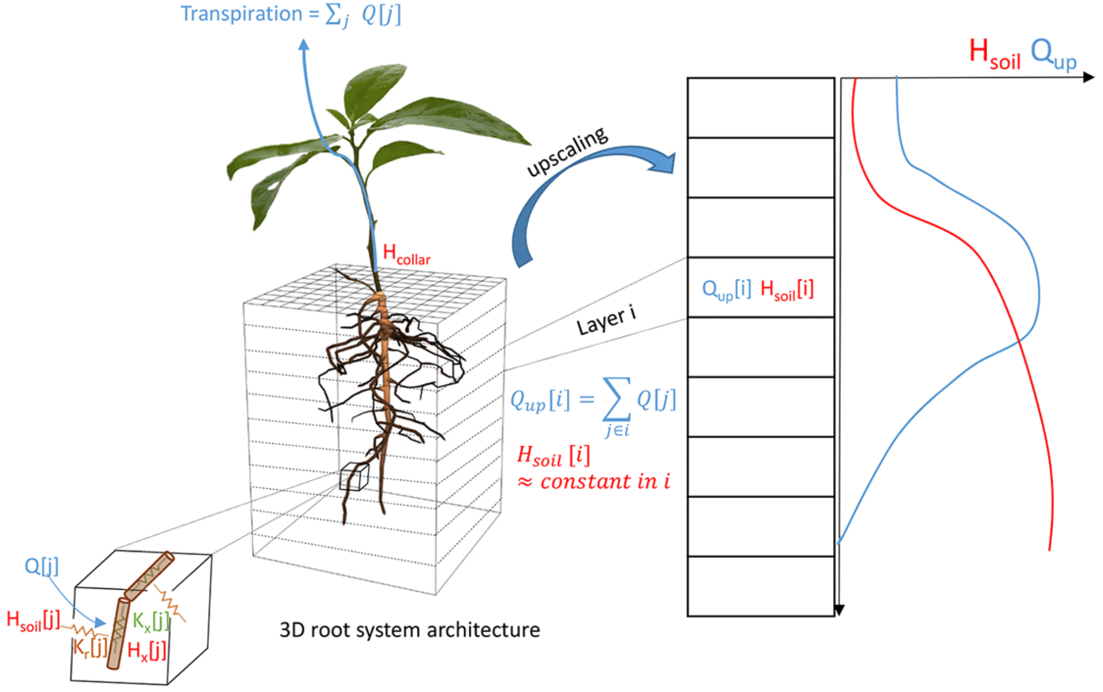
Modeling root water uptake

Root water uptake (1D case)

Recall the conservation of mass:

$$\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial z} - \underbrace{Q(z)}_{\text{Sink term}}$$

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[K_z\left(\frac{\partial h}{\partial z} + 1\right)\right] - Q(z)$$



Vanderborght et al. (2021), HESS

Modeling root water uptake

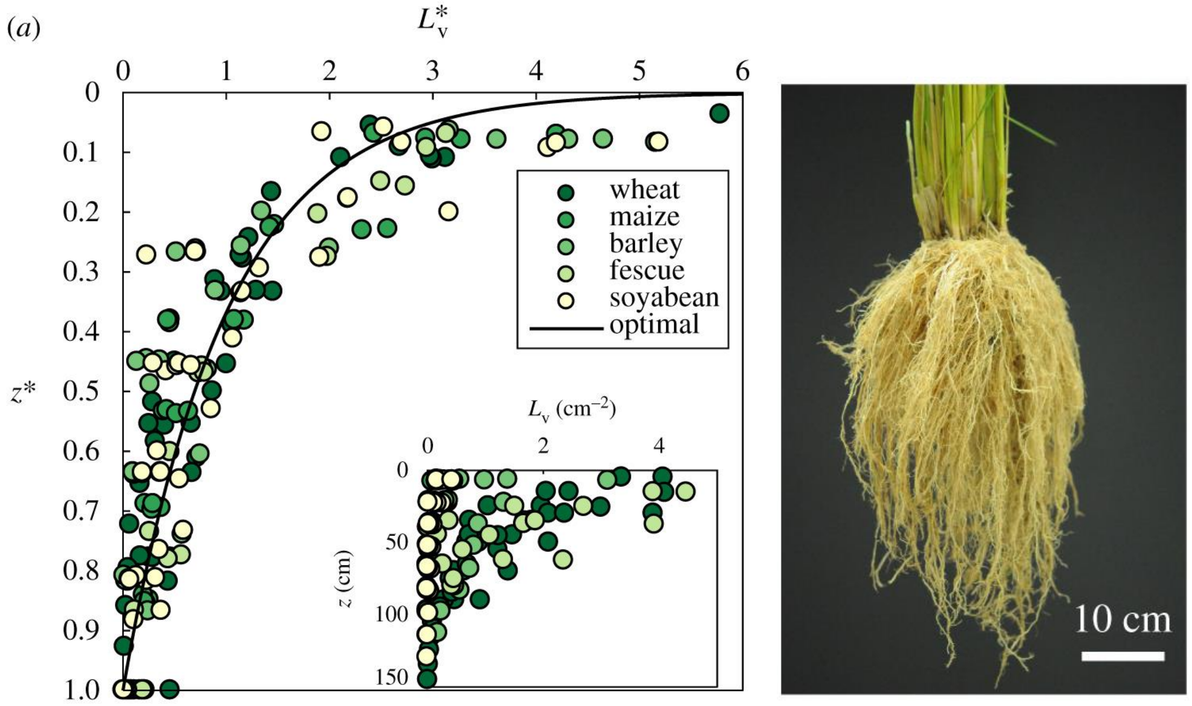
Root water uptake (1D case)

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[K_z\left(\frac{\partial h}{\partial z} + 1\right)\right] - Q(z)$$

If the transpiration flux is known, the root water uptake term can be written as:

$$Q(z) = \frac{L_r(z)}{\int_{-z_r}^{0} L_r(z) \cdot dz} \cdot T_r$$

Where $L_r(z)$ is the root lenght density [cm cm^{-3}] at depth z



Normalized root length density distribution of various plants as a function of normalized depth (left). An example of plant roots (right).
Source: Jung et al. (2019) PRSI

Modeling root water uptake

EPFL

Root water uptake (1D case)

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[K_z\left(\frac{\partial h}{\partial z} + 1\right)\right] - Q(z)$$

If the transpiration flux is known, the root water uptake term can be written as:

$$Q(z) = \alpha_{rw}(h) \cdot \frac{L_r(z)}{\int_{-z_r}^{0} L_r(z) \cdot dz} \cdot T_r$$

Water stress coefficient
(depending on the soil water potential)

Potential
Transpiration

Water Availability

Atmospheric demand

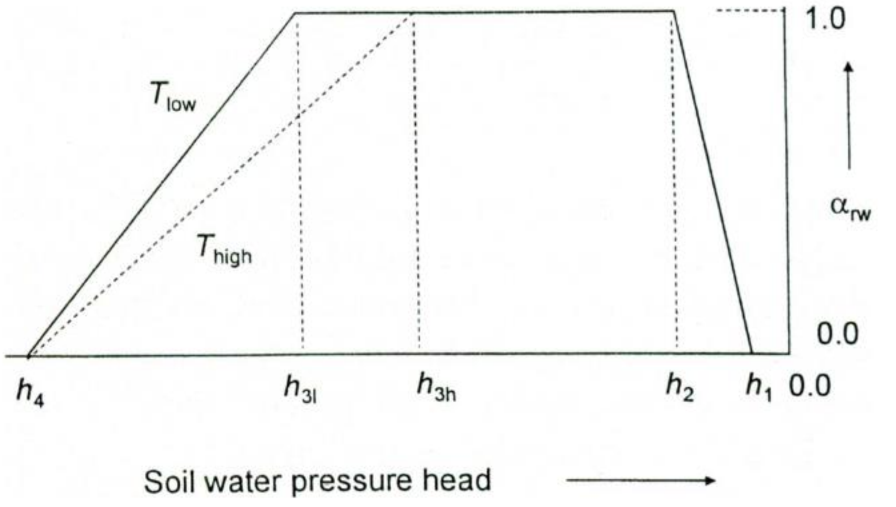


FIG. 1. Reduction coefficient for root water uptake, α_{rw}, as function of soil water pressure head h (cm) and potential transpiration rate T_p (cm/day^{-1}) (after Feddes et al. 1978). Water uptake above h_1 (oxygen deficiency) and below h_4 (wilting point) is set to zero. Between h_2 and h_3 (reduction point) water uptake is maximal. The value of h_3 varies with the potential transpiration rate T_p.

Feddes et al. (2001)

Modeling root water uptake

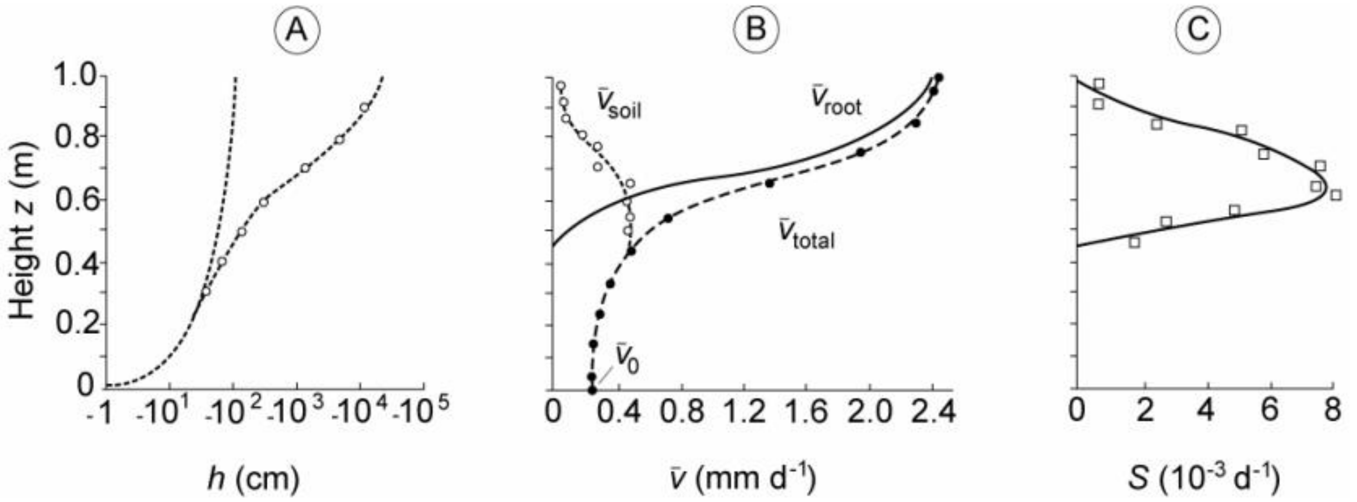


Figure 2. Profiles of time-averaged pressure head h (A); time-averaged cumulative water withdrawal of both a cabbage crop and clay soil $\bar{v}_{total}$, of soil only $\bar{v}_{soil}$ and of roots only $\bar{v}_{root}$ (B); time-averaged extraction rate S (C); for red cabbage on clay for the period of 18 through 25 July 1967 (after Feddes 1971)

See: https://edepot.wur.nl/35358

Modeling root water uptake

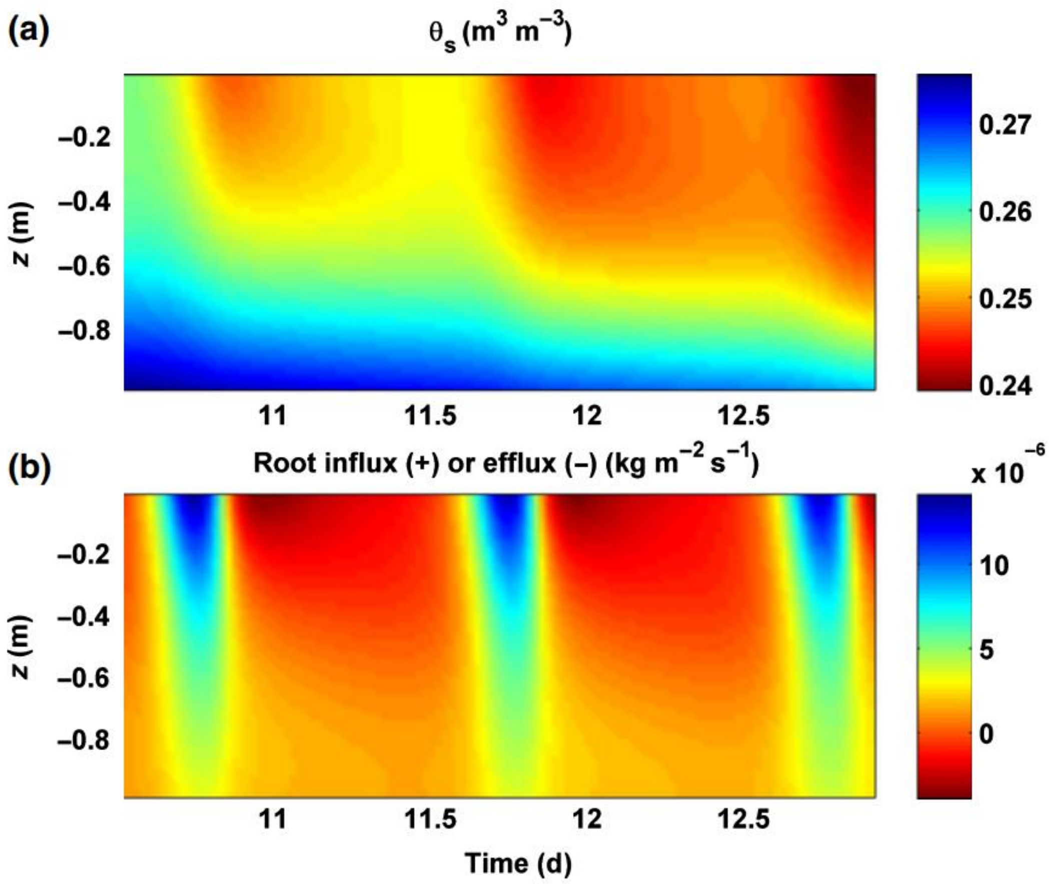


Fig. 5 Modeled profiles of (a) soil water content (θ_s) and (b) root water influx (Q_r^+) or efflux (Q_r^-) on a per unit ground area basis for S6 (see Table 2 for model set-up).

Huang et al. (2017), New Phyt

This week exercises & assignments

- **Exercises** for **Weeks 6-7** are available in Moodle
- **General solution for steady unsaturated flow** (see pdf in Moodle)
- **Computer Lab:** Q&A (optional)

- Optional reading: Modeling Root Water Uptake (journal article)

Appendix

Ex.3.3 - Derivation of the Penman equation

We start by writing E and H using their aerodynamic formulations, and assuming that $r_{av} = r_{ah}$. In doing so, we also convert q to e, using the fact that :

$$q_a \simeq \frac{\epsilon e_a}{p_a} \tag{1}$$

For a saturated surface ($e_0 = e_s(T_0)$), this leads to

$$H = \rho_a \, c_p \, \frac{(T_0 - T_a)}{r_a} \tag{2}$$

$$E = \rho_a \, \frac{\epsilon}{p_a} \, \frac{(e_s(T_0) - e_a)}{r_a} \tag{3}$$

$$\tag{4}$$

E can be written as

$$E = \rho_a \, \frac{\epsilon}{p_a} \, \frac{(e_s(T_0) - e_s(T_a) + e_s(T_a) - e_a)}{r_a} \tag{5}$$

We introduce the drying power E_A :

$$E_A = \frac{\rho_a \, \epsilon}{p_a \, r_a} \, (e_s(T_a) - e_a) \tag{6}$$

Note that r_a depends on horizontal wind speed. It follows that :

$$E = \rho_a \, \frac{\epsilon}{p_a} \, \frac{(e_s(T_0) - e_s(T_a))}{r_a} + E_A \tag{7}$$

The next step is to linearize $e_s(T_0)$ using a first order limited development around T_a, which is close to T_0 :

$$e_s(T_0) \simeq e_s(T_a) + e_s'(T_a) \, (T_0 - T_a) = e_s(T_a) + \Delta \, (T_0 - T_a) \tag{8}$$

This leads to

$$E = \rho_a \, \frac{\epsilon}{p_a} \, \frac{\Delta \, (T_0 - T_a)}{r_a} + E_A \tag{9}$$

We then use the relation between $(T_0 - T_a)$ and H :

$$E = \frac{\epsilon \, \Delta}{c_p \, p_a} \, H + E_A \tag{10}$$

and the energy budget equation :

$$R_n = H + LE + G \tag{11}$$

The downward heat flux G into the soil (counted here positively if the flux cools the surface) exhibits a very strong diurnal cycle, with values that can exceed 100 W.m^{-2} at noon, and conversely very negative values (warming the surface) at night. This flux depends a lot on surface temperature, which we want to eliminate. It can be neglected if we work on average over one day, or any multiple of one day. **This is a very important validity condition for the Penman equation**. In such conditions, the surface energy budget can be simplified as :

$$R_n = H + LE \tag{12}$$

so that

$$E = \frac{\epsilon \, \Delta}{c_p \, p_a} \, (R_n - LE) + E_A \tag{13}$$

Appendix

Introducing the psychrometric "constant" $\gamma = (c_p\, p_a)/(\epsilon L)$, we get

$$\frac{\epsilon\,\Delta}{c_p\, p_a} = \frac{\Delta}{\gamma\, L} \tag{14}$$

$$E = \frac{\Delta}{\gamma}\left(\frac{R_n}{L} - E\right) + E_A \tag{15}$$

Re-arranging this expression to isolate E gives the Penman equation :

$$E = \frac{\Delta}{\Delta+\gamma}\,\frac{R_n}{L} + \frac{\gamma}{\Delta+\gamma}\,E_A \tag{16}$$

Remark 1 : the above development assumes that R_n is independent from T_0, which is far from true, since it includes the upward long-wave radiation from the surface :

$$R_{lu} = \varepsilon_s \sigma T_0^4 \tag{17}$$

Most often, this term is estimated using T_a instead of T_0. In particular, it is the case in the FAO report on crop evaporation. A better approximation could be a achieved using the first-order limited development around T_a :

$$T_0^4 \simeq T_a^4 + 4T_a^3(T_0 - T_a) \tag{18}$$

Introducing the net short-wave radiation $R_{sn} = (1-a_s)R_{sd}$, the energy budget equation can be rewritten as

$$R_{sn} + \varepsilon_s \sigma T_a^3 (T_0 - T_a) = \rho_a\, c_p\, \frac{(T_0 - T_a)}{r_a} + LE \tag{19}$$

From this, we can find $(T_0 - T_a)$ as a function of R_{sn}, T_a, and E, then proceed from Eq. 9.

Remark 2 : to derive the Penman-Monteith (1965) equation for unstressed vegetation, we follow the same sequence as for the Penman equation, but we use the following initial expression of E, depending on the minimum stomatal resistance r_0 :

$$E = \rho_a\, \frac{\epsilon}{p_a}\, \frac{(e_s(T_0) - e_a)}{r_a + r_0} \tag{20}$$

what leads to

$$E = \frac{\Delta\, \frac{R_n}{L} + \gamma\, E_A}{\Delta + \gamma\left(\frac{r_a + r_0}{r_a}\right)} \tag{21}$$

The FAO report from Allen et al. (1986) defines the reference ET, ET_0, from the Penman-Monteith equation, with $r_a = 208\, /\overline{u}(z = 2m)$ and $r_0 = 70$ s.m^{-1}

Actual ET can also be estimated by the the Penman-Monteith equation, by accounting the effects of environmental stresses and vegetation properties (albedo, physiology (r_0), LAI, height and roughness) owing to appropriate resistance formulations.